

Substrate

DNA/RNA are a chain of 4 kinds of bases A , G , C , T/U .

They may *hybridize* with each other primarily by 3 types of base pairs (shown left) between purines (A, G) and pyrimidines (C, T/U).

-- AcGucAu ->

 \leftarrow Ugogguy

-- AcGucAu -> >- U GC GG U A --

Benenson et al. Nature 414, pp.430-434, 2001 7

Helix

Atoms of single-stranded RNA structures

Enzyme-driven single-stranded computation

A molecular SAT solver.

- 1. Literals are assigned with DNA sequences in such a way that
	- x and $\neg y$ hybridize with each other iff $x = y$
	- x and y never hybridize, or neither do $\neg x$ and $\neg y$
- 2. A SAT instance is programmed as a pool made of DNA sequences obtained by choosing one literal from each clause and catenating the corresponding DNA sequences.
- 3. Such a DNA sequence forms a hairpin iff both x and $\neg x$ of a variable are involved.
- 4. A restriction enzyme cuts a hairpin.
- 5. It suffices to check if some DNA sequence has "survived."

Sakamoto et al. Science 288, pp.1223-1226, 2000

RNA co-transcriptionality

Programmable platform for in vitro/vivo computations

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Geary, Rothemund, and Andersen, Science 345 (6198): 799-804, 2014 12

RNA polymerase

Figure 7-7 Essential Cell Biology (©Garland Science 2010)

RNA polymerase

The idea of this diagram is from Feynman Lectures on Computation, 1996

Transcripts from a single template in parallel :

End

DNA

Begin

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RNA origami

architecture for hard-coding a structure into CF

```
int main() {
   while (1) {
         std:cout << "Hello RNA World!!";
    }
   return 0;
}
```
Geary, Rothemund, and Andersen, *Science* 345 (6198): 799-804, 2014 16

Helix

co-axial stacking

Helices are stabilized **co-axially** via base-stacking at their interface. Two common motifs involving co-axial stacking are:

RNA origami

modular design of an RNA tile

Geary, Rothemund, and Andersen, Science **345** (6198): 799-804, 2014

RNA origami

hard-coded CF

Helix

co-axially stacked into a viral backbone

A. van Belkum et al. Nucleic Acids Research 13(21), pp.7673-7686.

RNA sequences are capable of keeping their 3'-end away from their 5'-end, thus,

- folding into non-tree structures, transcending the bound of CFL;
- avoiding to be degraded by ribonuclease (RNase).

Helix

RNA triple helix

RNA Origami to Oritatami

model of CF-driven computing

model of CF-driven computing

The nascent fragment tries to fold with as many bonds as possible.

model of CF-driven computing

If there are more than one way of fixing H7 point-wise or bond-wise, then **nondeterminism**.

Geary et al. **MFCS2016**: 43:1-43:14 25

model of CF-driven computing

An oritatami system consists of

- A finite set Σ of types of abstract molecule (bead).
- $w \in \Sigma^*$ (transcript)
- $R \subseteq \Sigma \times \Sigma$ (affinity/binding rule)
- \bullet δ (delay)
- α (*arity*), max # of bonds per bead, formed on the first-come-first-served basis

States by definition!? No! Implement them if needed.

Glider, a self-standing motif

With $\delta = 3$, arbitrary arity $\alpha \ge 1$, $R = \{(a, a'), (b, b')\}$, the periodic transcript a-•-b'-b-• $a'-a--$ folds into the self-standing glider motif.

context-sensitive folding

A transcript can fold differently, depending on what are around (environment).

Watters et al. Nat. Struct. Mol. Biol. 23(12), pp.1124-1131, 2016 29

context-sensitive folding

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context-sensitive folding

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 $(F2)$

(FO)

(F1)

 $(F3)$

 $(F4)$

(F5)

Complex structures are often made of simple identical (homo-) units (polymers) in nature.

A periodic RNA transcript (homo-polymer) can be transcribed from a circular DNA.

Homo-polymeric CF-driven computing

Zigzag binary counter

- The first oritatami implementation
- Fixed bit-width (3 in the right figure), but later endowed with capability to widen by 1 bit at every overflow
- A transcript is of period 60 as **0-1-…- 11-12-…-29-30-…-41-42-…-59-0-1-…**.
- Increment by 1 per zigzag.
- The factors **0-…-11** and **30-…-41** serve as a half-adder by folding into one of 4 possible conformations, depending on what are around.

Homo-polymeric CF-driven computing

Turing-universality

- Cyclic tag system (cts) is a binary rewriting system made of a cyclic list of u_0 , u_1 , …, $u_{k-1} \in \{0, 1\}^*$ and a pointer $0 \leq p < k$. It rewrites a word $w = a_1 a_2 \cdots a_n$ as:
	- 1. temp = $a_2 \cdots a_n$ (a_1 is deleted from w)
	- 2. If $a_1 = 1$, temp = temp $\cdot u_p$
	- 3. $W =$ temp
	- 4. $p++$ (mod k)

Homo-polymeric CF-driven computing

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Tessellation by a transcript of period 37 at delay 3

Turedo (Tur[-ing] + [Ter-]edo [navilis]) Programming language for CF

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2D Turing machine on the hex grid that is self-avoiding, that is,

- Once visited, a cell won't be visited again.
- According to the configuration within the radius-^r from its head, it colors the current cell and decides which neighbor to visit next, where r is a system parameter.

Example of radius-1 Turedo (mod-4 clockwise walker)

2D Turing machine on the hex grid that is self-avoiding, that is,

- Once visited, a cell won't be visited again.
- According to the configuration within the radius- r from its head, it colors the current cell and decides which neighbor to visit next, where r is a system parameter.

- A radius-1 stateless Turedo $T = (A, \delta)$ is a pair of a tape alphabet A including the blank ⊥ and a transition function δ : $A^6 \rightarrow A \times \{BR, FR, S, FL, BL\}$.
- Suppose T has come from north, where x_5 is written. If $\delta(q, x_0, x_1, x_2, x_3, x_4, x_5) = (a, d)$, then it
	- 1. writes $a \in A$ at the current cell, and
	- 2. moves to the neighbor cell in the direction d , as long as the cell is still blank; otherwise, it halts.

Turedo-to-Oritatami Compiler Theorem.

A radius-1 stateless Turedo $T = (A, \delta)$ can be programmed into a transcript w of period $O(|A|^6 \log |A|)$ with which the deterministic delay-3 oritatami system (Σ , w, R, 3, 6 simulates *intrinsically, where*

- Σ is universal, that is, independent of T, and consists of 1753 bead types.
- R is also universal.
- Each period of w folds into a **macrocell** of side length $O(|A|^3 \log |A|)$.

Macrocell

- 1. Scaffold layer
- 2. Read $(log|A|$ bits/side) layer
- 3. Write (log|A| bits/side) layer
- 4. Exit layer

 $O(|A|^3 \log |A$

Shift-driven computing

Period of transcript

A Turedo $T = (A, \delta)$ is encoded in the period of a transcript as:

Scaffold → **Read** → **Write** → **Speedbump** → **Exit** →

Scaffold hardcodes the macrocell's skeleton

Read $x = (x_0, x_1, x_2, x_3, x_4, x_5)$ with $x_s = (b_{s,0}, b_{s,1}, ..., b_{s,log|A|-1})$

• Weigh-sums the bits in $x_0, x_1, ... x_5$ in this order as:

$$
\Delta(x) = \sum_{s=0}^{5} \sum_{i=0}^{\log|A|-1} b_{s,i} 2^{s \log|A|+i}
$$

Pushes the remaining transcript forward by this offset.

NOTE. these bits must have been written in a **uniform** format. It is uncomputable whether a cell will be visited; let alone from which direction it will be entered.

 x_{5}

 x_{2}

 x_4

 x_3

 x_0

 x_1

Read **U:** Reading pockets

OK, readers, it's you who weighs bits!!

1 Otherwise

Read **U:** Reading pockets

If the *j*-th bit is 1, then $\Delta(x)$ += 2^{slog[A]+j}

Period of transcript

A Turedo $T = (A, \delta)$ is encoded in the period of a transcript as:

Scaffold → **Read** → **Write** → **Speedbump** → **Exit** →

Write a and d (letter & direction)

- All the transition tables for each bit and for exit-direction are encoded: $|A|^6$ entries per table.
- This layer is shifted by $\Delta(x)$ so that only the referred entry by x of each table is exposed at a position readable later while the others are "hidden."
- Outputs must be in a uniform format along all the sides.

Speedbump absorbs Δ

Exit at the side $d(x)$ specified by **Write**

 x_{5}

 $\pmb{x_2}$

 x_4

 x_3

 x_0

 \boldsymbol{x}_1

Read **U** to Write **U**: U-turn pocket

The shift is transferred from **Read** to **Write**.

Write \mathbf{U} : tables shifted by $\Delta(x)$

NOTE: The system knows in the stage of programming tables for each x , whether each bit will be covered by **Exit** according to the exit direction $d(x)$.

"Foldable" Speedbump: absorbing $\Delta(x)$

Based on the straight speedbump [PchelinaSSU20] but quadratically more spaceefficient.

,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,

RNA Spinner

In vitro/vivo auto-synthesis of RNA components by NFA

 $uxs\ell\theta(s) yv \rightarrow uv$ if

Co-transcriptional splicing

- 1. (x, y) is an enzymatically-recognizable context and
- 2. s $\ell\theta(s)$ is a *stable* hairpin, where θ is an antimorphic involution.

An NFA for a superset of .

RNA spinner

 $\frac{1}{\sqrt{2}}$

Circular

DNA

 $\mathsf{P}^{\;\prime}$

0
}
{,

∙

 $\mathbf{\Omega}$

Co-transcriptional

splicing

∙

a

∙

encoding

 \mathbf{p}

∙

Any molecular system consists of *finitely many* $(kinds)$ of DNA and RNA sequences. Let R be the *finite* set of RNA sequences it involves.

Recall that RNAs are naturally degraded.

 $y_0 a \overrightarrow{x_0} \cdot y_0 bx_1 \cdot y_1 ax_1 \cdot y_1 bx_0 - \overleftarrow{y_0} a x_0 \cdot y_0 bx_1 \cdots$ $y_0 a a \overrightarrow{x_0} \cdot \overrightarrow{y_0} b x_1 \cdot y_1 a x_1 \cdots$ $y_0 a a b x_1 \cdot y_1 a x_1 \cdots$

Problems to be solved for RNA Spinner

Problem 1.

Given R and a finite set D of domains (via which sequences in R interact with each other or with other molecules), construct an NFA A with as few transitions as possible s.t.

$R \subseteq L(A) \subseteq R \cup \overline{\Sigma^* D \Sigma^*}$

By setting $D = \Sigma$, this problem is reduced to the NP-hard problem of finding a transition-minimal NFA for finite languages [GruberH07]. Such a ubiquitous domain however turns any system into a chaotic soup.

Problems to be solved for RNA Spinner

A hairpin gets less stable with a longer loop and a shorter stem.

Contribution by stem is linear, while it remains open how a loop is penalized.

Problem 2

Propose a proper energy model for RNA hairpin stability, and study hairpin-related operations by considering only stable hairpins in the model.

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UFC

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